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Universality of a dynamical percolative approach to $1/f^{\gamma}$ noise

M. Celasco[a] and R. Eggenhöffner

Dipartimento di Fisica, Università di Genova, via Dodecaneso 33, 16146 Genova, Italy

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Abstract. A dynamical percolative model explaining the universality of $1/f^{\gamma}$ noise is reported. Exponents γ ranging from 0 to 2 are obtained under the hypothesis that noise originates from random switching events between two ON–OFF states in elemental parts (*switchers*) of a physical system. The usual noise behaviour with γ very close to 1 in an arbitrarily wide frequency range is obtained assuming a statistical distribution of switcher relaxation time τ proportional to τ^{-1}, as in McWhorter's model. The impact of these results with respect to recent self-organised criticality models is discussed.

PACS. 74.40.+k Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.) – 64.60.Ak Renormalization-group, fractal, and percolation studies of phase transitions – 05.40.Ca Noise – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion

1 Introduction

The astonishing appearance of $1/f$ noise in a large number of different phenomena is a very intriguing problem, which till now has not been completely understood. One of the most ambitious objectives is surely the determination of a "universal" law, able to explain the rise of $1/f$ noise from elemental events. In such a way, the problem would be reduced to identify such events in each phenomenon. Actually, signals with $1/f$ power spectra were obtained with theoretical models and simulations; however, the universality of these models is still lacking, since models are often unrealistic or applicable only to delimited problems.

In the present work, we introduce a dynamical percolative model, where $1/f^{\gamma}$ noise is simply recognised in the evolution of a network of elemental objects (*switchers*) randomly jumping between two states, ON and OFF, ruled by a time dependent probability distribution. We show that the general $1/f^{\gamma}$ behaviour with the exponent γ continuously varying between 0 and 2 is strictly related to the jumping probabilities of the switchers and that it holds in a wide frequency range (typically two frequency decades). The usual $1/f$ noise behaviour with slope very close to 1 in an arbitrarily wide frequency range is obtained assuming a statistical distribution of switcher relaxation time τ proportional to τ^{-1}.

Our approach is entirely general and it can be applied to any physical system obeying a linear relation between an external constant input and the output of the system, where the time dependent transfer function results from the combined behaviour of several *switchers*. Examples are the voltage fluctuations of granular superconductors as well as trapping-detrapping mechanisms in semiconductors, but extensions to many natural phenomena are straightforward.

[a] celasco@fisica.unige.it

2 1/f noise in recent models

Recent works trying to explain the $1/f$ noise from elemental and universal mechanisms can be grouped in three different research approaches: 1) the sandpile model or self-organised criticality (SOC); 2) the percolative-like approach and 3) the avalanche model and clustering transition phenomena.

1) Starting from the pioneering work of Bak, Tang and Wiesenfeld [1] based on the criticality conditions of a sandpile in slow motion or an ensemble of coupled oscillators, a dissipative SOC model was developed [2,3] and power spectra with $1/f$ trend in a wide frequency range (four decades) were obtained in 1D and 2D.
2) In percolative models [4–8] two-phase systems are considered. Varying the fraction of one phase with respect to the other, the fluctuations of a physical quantity related to the two phases are computed for different system configurations and are treated as the intensity of "noise" related to the physical quantity. The time evolution of these systems, however, was seldom considered [9] and spectral noise structure was not investigated.
3) Avalanche [10–12] and clustering [13] models were introduced to explain the displacements of particles starting from initial localised non-equilibrium instabilities which propagate through the interior of a sandpile-like column as in the SOC model. Equivalently, these phenomena are related to sequences of

clusters of elemental events or pulses. Both the sandpile and the avalanche models, under suitable conditions, can be recognised as extensions of percolative-type transitions since the stable configurations of the dynamical systems are similar to the critical point of the percolative framework [1].

Avalanche models give rise typically to $1/f^{\gamma}$ spectra with $1.5 \leq \gamma \leq 2$ in a wide frequency range in agreement with experimental results (superconducting vortex [14], Barkhausen effect [15], vacancy and dislocation dynamics in metals [16]), whereas models basically derived from SOC lead typically to $\gamma \approx 1$ and, in the limit without dissipation, to a flat spectrum in a wide range at low frequencies and to $\gamma \approx 1.5$ at high frequencies [2].

Because of the boundary for γ respectively below and above a critical value of about 1.5, both SOC and avalanche models cannot attain the character of universality invoked in the above approaches.

3 The dynamical percolative model

In order to overcome the above discrepancies, in the present paper, a dynamical percolative model is developed to describe the fluctuations of the generalised resistance $R(t)$ of a physical system composed of many elemental objects randomly jumping between two states, ON and OFF. The model gives constant power spectral density $S_R(f)$ at low frequencies and a f^{-2} law at high frequencies: in this way, the crucial requirement of the convergence of the power spectrum is satisfied. The transition from $S_R(f) \sim$ const. at low f to $S_R(f) \sim f^{-2}$ at high f is smooth, and it is possible to make it fit with a $S_R(f) \sim f^{-\gamma}$ law, with $0 \leq \gamma \leq 2$ on a limited frequency range. We will show that with a further simple hypothesis an arbitrarily wide frequency range with $S_R(f) \sim f^{-1}$ is obtained. Actually, the subsequent percolative analysis can be extended to any physical system obeying a linear relation $V(t) = R(t)I$ between an external constant input I and its output $V(t)$, where the transfer function $R(t)$ results from the combined behaviour of several independent resistor-like objects. In the present model, the transitions of elemental links in a 3D network are viewed as the source of noise. Unlike other percolative models, where links are assumed to fluctuate, our links behave as switches, which abruptly change their generalised resistance $r(t)$ jumping between two well-defined ON and OFF states. These objects are to be named switchers, to distinguish them from links usually considered in other percolative models. The ON state corresponds to a shorted switcher ($r = 0$), while the OFF state corresponds to a switcher with a finite resistance r. For simplicity, we assume that the value of the resistance in the OFF state is the same for each switcher. Thus, each switcher acts as a random telegraph and the combination of switchers gives rise to fluctuations of the network resistance $R(t)$.

In order to quantify this noise, we introduce the percolative parameter p and the dynamical parameter q, which governs the time evolution of a single switcher. The parameter p represents the probability that a switcher is in the ON state at time t, whereas q is the probability that the same switcher is in the OFF state at time $t + \Delta t$, if it is ON at time t. The definition of q depends on the length of the interval Δt. We assume that Δt is sufficiently small to neglect multiple link switching. Both p and q are supposed time independent. The autocorrelation function $C_r(t) \sim \mathrm{e}^{-t/\tau}$ of a switcher resistance $r(t)$ can be computed [17] by Kolmogorov equations:

$$\frac{\mathrm{d}P_{ij}}{\mathrm{d}t}(t) = \sum_k P_{ik}(t) \frac{\mathrm{d}P_{kj}}{\mathrm{d}t}(0) \tag{1}$$

where $P_{ij}(t)$ are the transition probabilities between the ON and OFF states, as discussed in details in the Appendix of reference [17]. The probabilities that a signal remains unchanged after the short time interval Δt are P_{11} and P_{00} whereas P_{01} and P_{10} give the transition probabilities between the two states in the same time interval. As a result, the decay time τ of the autocorrelation function $C_r(t)$ turns out to be:

$$\tau = \frac{(1-p)\Delta t}{q} \,. \tag{2}$$

If two opposite boundaries of the 3D network are connected to a constant current source I, our purpose is to evaluate the fluctuations of the voltage $V(t)$ between the two terminals produced by the jumping of switchers, or, equivalently, the time evolution of the network resistance $R(t)$.

If the fraction p of switchers with $r = 0$ is very large, we can always find an ON path from one extremity of the network to the other; in such case $R = 0$. On the contrary, if p is low enough, such an ON path does not exist, so that the equivalent resistance R differs from zero. This fact introduces necessarily a percolative aspect in our model. From percolation theory, it is well known that, if the size of the network is infinite, a well-defined value of $p = p_{\mathrm{c}}$ appears; for $p < p_{\mathrm{c}}$ the network is resistive, whilst for $p > p_{\mathrm{c}}$ the network resistance is zero. The percolative threshold p_{c} can be evaluated by numerical computations for several systems: in the case of a simple cubic network, $p_{\mathrm{c}} = 0.25$. The behaviour of the equivalent resistance R near the percolative threshold is $R \sim (p_{\mathrm{c}} - p)^s$, with the critical exponent $s = 0.76$ (for a simple cubic network) [18].

In order to obtain the time evolution of a system of switchers for an adequate number of p and q values, a cubic network of 3×10^3 links was considered, whose resistances evolve in the random telegraphic way discussed above. For each configuration of the system, the network resistance R was computed by means of a transfer matrix method [19].

4 Results and discussions

In Figure 1, the effect of the dynamical parameter q is shown: in the graphs in the first column, the dynamical parameter $q = 0.15$ and, in the second column, $q = 0.015$.

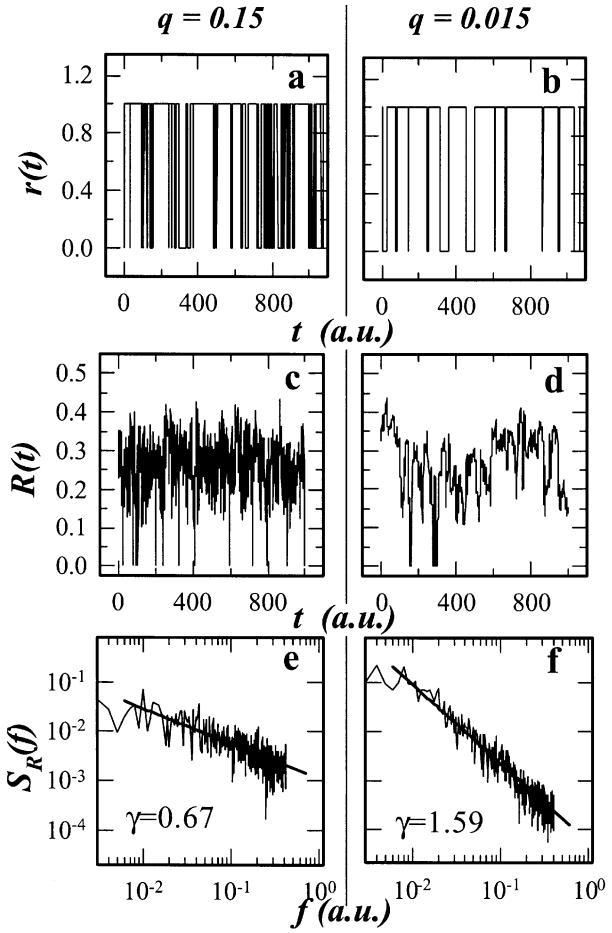

Fig. 1. Effect of dynamical parameter q on $r(t)$ (single switcher resistance), on $R(t)$ (whole network resistance) and on $S_R(f)$ (power spectral density). In the first column, $q = 0.15$; in the second column, $q = 0.015$. The percolative parameter p is fixed at 0.2. All the reported properties are in arbitrary units. In (e) and (f), the best-fit linear behaviours are also shown.

In every case, $p = 0.2$. In Figures 1a *vs.* 1b the behaviour of the resistance $r(t)$ of a single switcher, randomly chosen inside the network, is reported, and in Figures 1c *vs.* 1d the equivalent resistance of the whole network, $R(t)$, is reported. In Figures 1e *vs.* 1f, the power spectral densities $S_R(f)$ of signals presented in Figures 1c *vs.* 1d are shown. Each spectrum is comprised between $f = 1/N\Delta t$ and $1/2\Delta t$, where $N\Delta t$ is the total length of the simulation.

The combination of several switching events gives rise to small jumps of the system resistance $R(t)$. For a large number of switchers and an interval Δt small enough, these jumps are actually fluctuations of $R(t)$. The parameter q influences the rate of fluctuation of the network; the fluctuations become slower and more correlated as q decreases. $S_R(t)$ behaves as $f^{-\gamma}$ with $\gamma = 0.67$ ($q = 0.15$) and $\gamma = 1.59$ ($q = 0.015$). The log-log behaviour of the power spectral density was linearly fitted in two decades frequency range to obtain the parameter γ values by means of the facilities provided by MATLAB code [20]. The errors in the calculations were estimated as small as 0.01–0.03, *i.e.* negligible for the following discussions and similar to typical experimental uncertainties in the estimates of the parameter γ [21].

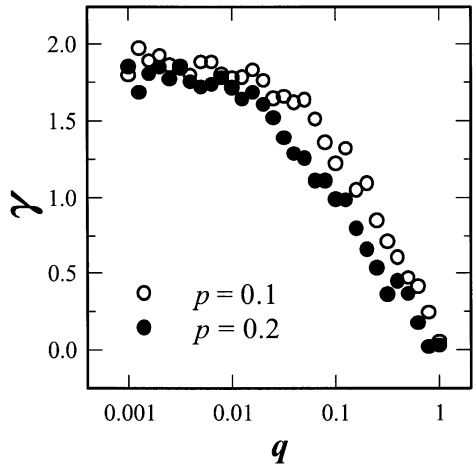

Fig. 2. Dependence of the noise exponent γ on the dynamical parameter q at $p = 0.1$ (○) and $p = 0.2$ (●).

Numerical simulations with $N = 10^3$ and different values of p and q were performed. In every case, γ decreases at increasing the dynamical parameter q; γ is always comprised in the range $0 \leq \gamma \leq 2$ (Fig. 2).

Thus, with the present percolative model and within the simplest assumption to attribute the same values of the RTS parameters to all the junctions in the network, the fundamental result that the coefficient γ ranges from 0 up to 2 is obtained. At high q, γ tends to zero, a reasonable result since fluctuations behave as white noise when the switching rates are high. At low q, γ tends to the constant value 2, consistently with avalanche models [10,15,16]: when q is low, the fluctuations of the network resistance $R(t)$ are essentially due to single isolated events characterised by large τ. The increase of γ with dissipation observed in SOC models [2] is also consistent with our results, since characteristic times τ become longer when dissipation increases.

From the definition of q and the results of Figure 2, it can be argued that $S_R(f)$ on many decades has the asymptotic behaviour

$$S_R(f) \sim \begin{cases} \text{const.} & \text{at low frequencies} \\ f^{-2} & \text{at high frequencies.} \end{cases} \tag{3}$$

The transition from the constant regime at low frequencies to f^{-2} behaviour at high frequencies should be so smooth as to make possible the approximation $S_R(f) \sim f^{-\gamma}$ on few decades, as reported in Figures 1e and 1f.

The experimental observation that many systems behaves $1/f^{\gamma}$ with γ close to 1 in a wide frequency range [22] is explained by the present model with the further hypothesis that the parameter q is spatially distributed over the switchers. In particular, for a fixed value of Δt, a distribution of q between $q_{\min}$ and $q_{\max}$ with a statistical weight $g(q) \sim q^{-1}$ is assumed. With respect to the calculations giving the results reported in Figure 2 in which $\gamma = \gamma(q)$

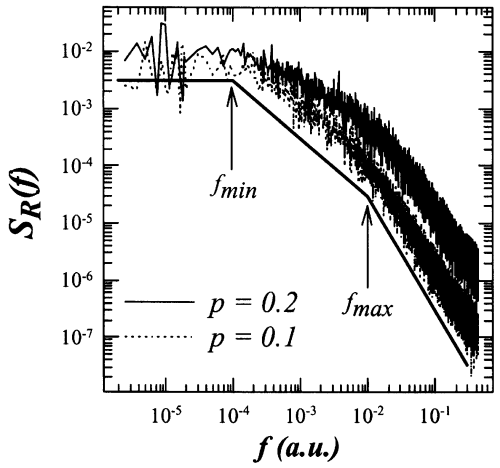

Fig. 3. Power density spectral noise (in arbitrary units) obtained assuming a distribution of q values between two values $q_{\min} = 5 \times 10^{-4}$ and $q_{\max} = 5 \times 10^{-2}$ with a statistical weight $g(q) \sim q^{-1}$. This choice gives a change in the frequency behaviour at the corner frequencies $f_{\min} \approx 10^{-4}$ and $f_{\max} \approx 10^{-2}$. Upper (solid) curve is obtained with $p = 0.2$, lower (dashed) curve with $p = 0.1$. Both spectra were obtained averaging eight independent simulations. The lines with $\gamma = 0$, $\gamma = 1$, $\gamma = 2$ at $f < f_{\min}$, $f_{\min} < f < f_{\max}$, $f > f_{\max}$, respectively, are reported shifted downward as eye-guide.

and q is kept constant over the network of switchers in each simulation consistently with ordered physical or natural systems, the latter assumption of allowing q to vary between two fixed limits is more appropriate, for instance, to study inhomogeneous systems. As a consequence of (2), where $\tau \propto q^{-1}$, our assumption corresponds to the statistical distribution of decay times $g(\tau) \sim \tau^{-1}$. The introduction of two limiting values of the dynamical parameter q and the relationship between q and τ distributions leads to two corner frequencies $f_{\min}$ and $f_{\max}$, as shown in Figure 3. It is interesting to observe that the corner frequencies are approximately equal to

$$f_{\min} = \frac{1}{2\pi\tau_{\max}}, \quad f_{\max} = \frac{1}{2\pi\tau_{\min}}, \tag{4}$$

where $\tau_{\min}$ and $\tau_{\max}$ are the extreme values of the decay time of the autocorrelation function $C_r(t)$. At small values of p ($p \approx 0.1$, lower curve in Fig. 3) γ assumes values around 0 in the lower frequency region, of 1 in the range $f_{\min} < f < f_{\max}$ and slightly lower than 2 in the high frequency range.

The width of the region with $S_R(f) \sim f^{-1}$ can be arbitrarily increased by changing values of $\tau_{\min}$ and $\tau_{\max}$, obtaining the $1/f$ behaviour between $f_{\min}$ and $f_{\max}$ in a suitably wide range depending upon the choice of the corresponding decay times. From our simulations with higher values of p ($p = 0.2$, upper curve in Fig. 3), closer to the percolative threshold, three ranges of linear dependence of the $S_R(f)$ with frequency are still found, but lower values of γ are obtained in the second and third range ($\gamma \approx 0.6$ and $\gamma \approx 1.77$, respectively). Results are summarised in Table 1 to show that three ranges of linear behaviours

Table 1. Estimates of the parameter γ in the three frequency ranges delimited by f_1 and f_2 at two selected values of the percolative parameter p.

	$f_1 < f$	$f_1 < f < f_2$	$f > f_2$
$p = 0.1$	0.13 ± 0.16	-0.99 ± 0.02	-1.92 ± 0.01
$p = 0.2$	0.04 ± 0.16	-0.68 ± 0.02	-1.77 ± 0.01

are actually obtained in the three frequency ranges. In the lower range where statistical errors are intrinsically larger, we can only claim that the expected result $\gamma = 0$ is consistent with the calculations.

Among the most typical cases that can be explained by the present dynamical percolative model, we indicate as examples voltage fluctuations induced by the flow of a constant current in a superconductor in a magnetic field [17], density variations of vacancies and dislocations [16] as well as trapping-detrapping conductivity of semiconductors [23], hopping conductivity mechanisms in thin films and ionic current through biological membranes. For instance, the temperature dependence of γ was shown by experiments on semiconducting GaN-based MODFETS devices: experiments show that γ decreases from 1.03 to 0.85 and from 1.0 to 0.9 when temperature decreases from 300 K to 250 K and from 220 K to 130 K, respectively [21]. At decreasing temperature, trapping of carriers at interfaces will lower the carrier concentration and it will enhance noise levels in a thermally activated process. As shown in Table 1, in our simulations this effect is reproduced at increasing the percolative parameter p slightly above 0.1, *i.e.* by driving the system towards the percolation limit (at $p = 0.25$ as discussed above). Direct evidence of the noise from modulations of the percolation path lengths (and thus of the percolative parameter p) was also reported in Si:H thin films [24]. Previous noise experiments in thin films showed lower slopes (lower γ) in the central frequency range [22,25,26]. Due to their intrinsic nature, films can be supposed closer to critical conditions concerning the percolative threshold. In other systems, as in GaAs-based diodes, however, tunnelling between traps was postulated to give carrier fluctuations and the $1/f$ noise, as in the well known McWhorter's approach [27]. Following the suggestions by Lust-Kakalios [9] and Surya *et al.*'s works [21,24], noise is related to widely non-uniform distribution of trapping times, related to the corresponding distribution of the depth of the energy wells of the traps. In our approach, a wide energy and trapping times distribution leads to a wide distribution of the dynamical parameter q: mainly, low values of the γ coefficient will be selected accordingly to Figure 2 and noise is high as observed experimentally even at relatively high frequencies.

5 Conclusions

Our percolative approach with the dynamical parameter q common to the network of switchers predicts signals with $1/f^{\gamma}$ in the $S_R(f)$ on two decades typically, where $\gamma(q)$

is comprised between 0 and 2 at q varying in the range 10^{-3}–1. With the further simple assumption that the dynamical parameter is distributed between two characteristics values of the dynamical parameter over the whole switcher network, $S_R(f)$ is $1/f$ with γ close to 1 on an arbitrarily large number of decades. Thus, the variety of experimental frequency behaviour of noise in transport phenomena between the extreme cases $\gamma = 2$ (Barkhausen, vacancies, dislocations) and $\gamma = 0$ (white noise), and also the striking case of γ close to 1 (biological membranes, semi and superconductors, as well as most of the many examples quoted above) is accounted for within the same framework developed in the present paper.

More specifically, our dynamical percolative model can be extended straight to every physical system whose elements exhibit two opposite randomly switching states ON and OFF as to any systems flashing between two different configurations [28].

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